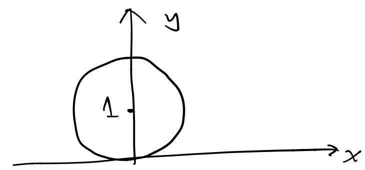
For 12th edition

Section 10.4



8. $x^2+y^2=2y, \quad y \ge 0$

$\Updownarrow$

$x^2+(y-1)^2=1$

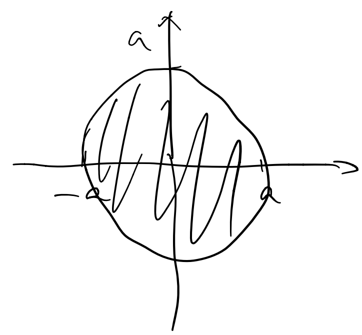
Let $x = r\cos\theta$. $\quad y = r\sin\theta$

$r^2 = 2r\sin\theta$.

or $\quad r = 2\sin\theta. \qquad \theta \in [0, \pi)$

Then the region is given by

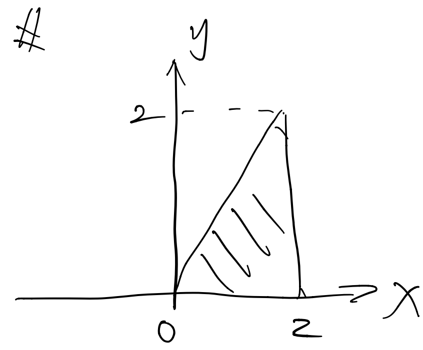
$\{0 \le r \le 2\sin\theta, \quad \theta \in [0,\pi)\}$ #



12 $\displaystyle\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx$

$\displaystyle = \int_0^a \int_0^{2\pi} r\,dr\,d\theta$

$\displaystyle = 2\pi \ \frac{1}{2}r^2\Big|_0^a = \pi a^2$ #

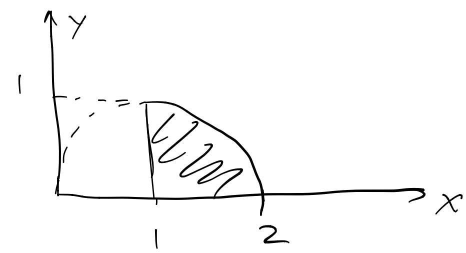


14. $\displaystyle\int_0^2 \int_0^x y\,dy\,dx$

$\displaystyle = \int_0^{\frac{\pi}{4}} \int_0^{\frac{2}{\cos\theta}} r\cdot\sin\theta\cdot r\,dr\,d\theta$

$\displaystyle = \int_0^{\frac{\pi}{4}} \frac{\sin\theta}{3}\cdot\left(\frac{2}{\cos\theta}\right)^3 d\theta = \frac{4}{3\cos^2\theta}\Big|_{\theta=0}^{\frac{\pi}{4}} = \frac{4}{3}$. #

22. $$\int_1^2 \int_0^{\sqrt{2x-x^2}} \frac{1}{(x^2+y^2)^2}\, dy\, dx$$



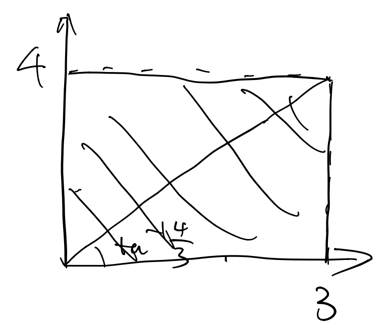
$$= \int_0^{\frac{\pi}{4}} \int_{1/\cos\theta}^{2\cos\theta} \frac{1}{r^4}\cdot r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{2}\left(\cos^2\theta - \frac{1}{4\cos^2\theta} \right) d\theta$$

$$= \frac{1}{8}\sin 2\theta + \frac{1}{4}\theta - \frac{1}{8}\tan\theta \Big|_0^{\frac{\pi}{4}} = \frac{1}{8} + \frac{\pi}{16} - \frac{1}{8} = \frac{\pi}{16}$$ #

26. $$\int_0^{\tan^{-1}\frac{4}{3}} \int_0^{3\sec\theta} r^7\, dr\, d\theta + \int_{\tan^{-1}\frac{4}{3}}^{\pi/2} \int_0^{4\csc\theta} r^7\, dr\, d\theta$$

$$= \int_0^4 \int_0^3 (x^2+y^2)^3\, dx\, dy$$



15.8

8. $$\iint_R 2(x-y)\, dx\, dy$$

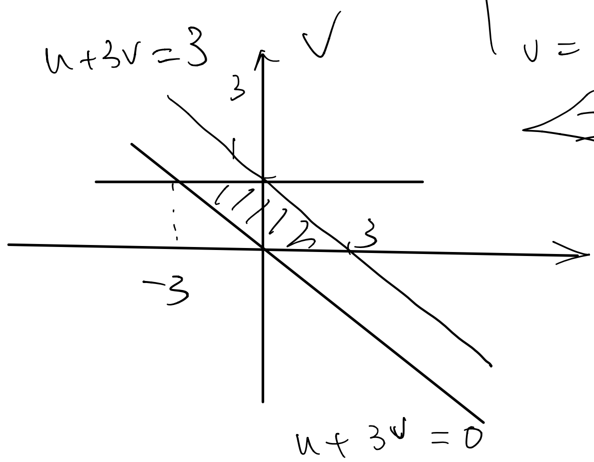
$$\begin{cases} u = 2x - 3y \\ v = -x + y \end{cases}$$

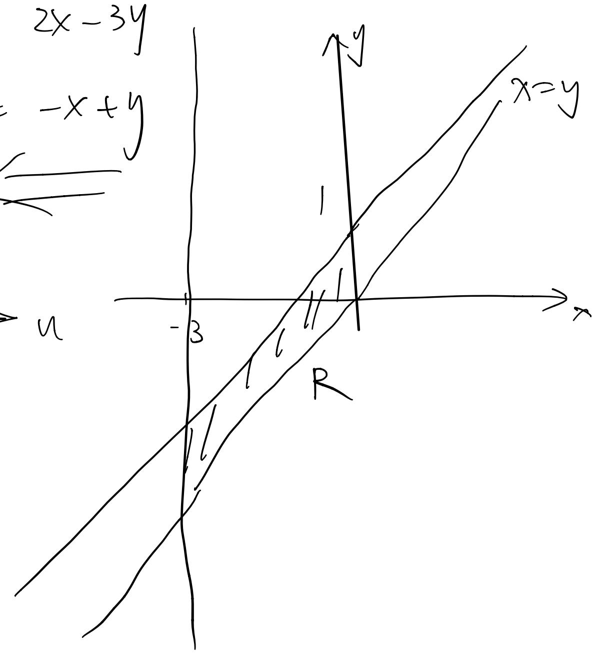
$\Updownarrow$

$$\begin{cases} x = -(u+3v) \\ y = -(u+2v) \end{cases}$$

$$\Rightarrow \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -1 & -3 \\ -1 & -2 \end{vmatrix} = -1$$

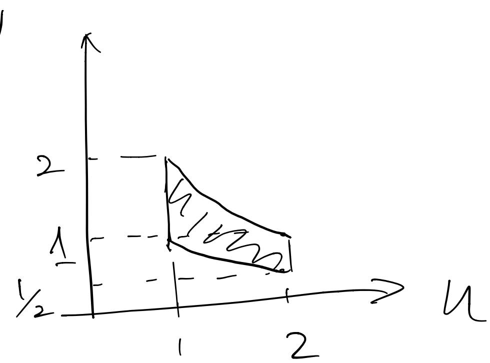
$$\begin{cases} u = 2x - 3y \\ v = -x + y \end{cases}$$





$$\iint_R 2(x-y)\,dx\,dy = \int_{-3}^{0} du \int_{-u/3}^{1} -2v \cdot 1\, dv + \int_{0}^{3} du \int_{0}^{1-u/3} -2v \cdot 1\, dv = -3$$

10a a. $J(u, v) = \begin{vmatrix} 1 & 0 \\ v & u \end{vmatrix} = u$



b. $\int_1^2 \int_1^2 \frac{y}{x}\, dy\, dx = \int_1^2 \int_{1/u}^{2/u} v \cdot u \, dv\, du$

$$\int_1^2 \frac{1}{x} \frac{y^2}{2}\Big|_{y=1}^{2} dx = \frac{3}{2} \cdot \log 2$$

$$\int_1^2 \frac{u v^2}{2}\Big|_{v=1/u}^{2/u} du = \int_1^2 \frac{3}{2} u^{-1} \, du = \frac{3}{2} \log 2$$

12. $\displaystyle \iint_{x^2/a^2 + y^2/b^2 \le 1} 1 \, dx dy = \iint_{u^2+v^2 \le 1} |ab| \, du dv = ab \cdot \pi$.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$$

14. $\displaystyle \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 1/2 \\ 0 & 1 \end{vmatrix} = 1$

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3 (2x-y) e^{(2x-y)^2} \, dx\, dy$$

$$= \int_0^2 \int_0^2 v^3 (2u) \, e^{(2u)^2} \cdot 1 \quad du dv$$

$$= \frac{1}{4} \, e^{(2u)^2} \Big|_{u=0}^{2} \times \frac{1}{4} v^4 \Big|_{v=0}^{2}$$

$$= e^{16} - 1 \quad \#$$

16 $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 2u & -2v \\ 2v & 2u \end{vmatrix} = 4(u^2+v^2)$$

$$\sqrt{x^2+y^2} = \sqrt{(u^2-v^2)^2+4u^2v^2} = u^2+v^2$$

$$\int_0^1 \int_0^{2\sqrt{1-x}} \sqrt{x^2+y^2}\, dy\, dx = \int_0^1 \int_0^u 4(u^2+v^2)^2 \, dv \, du$$

$$= \int_0^1 4u^5 + \frac{4}{5}u^5 + \frac{8}{3}u^5 \, du$$

$$= \frac{1}{6} \times \left(4 + \frac{4}{5} + \frac{8}{3}\right) = 56/45. \quad \#$$

22. Let $au = x$, $bv = y$. $cw = z$

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = abc$$

$$\text{Volume} = \iiint_{u^2+v^2+w^2 \le 1} abc \, du\,dv\,dw = \frac{4}{3}\pi abc.$$

24. $$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & 0 & 0 \\ y & x & 0 \\ 0 & 0 & 3 \end{vmatrix} = 3x = 3u$$

$$\Rightarrow \frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{1}{3u}$$

$$\iiint_D (x^2y + 3xyz)\,dx\,dy\,dz$$

$$= \int_1^2\int_0^2\int_0^3 (u \cdot v + vw) \cdot \frac{1}{3u}\,dw\,dv\,du$$

$$= \int_1^2\int_0^2\int_0^3 \left(\frac{1}{3}v + \frac{vw}{3u}\right)dw\,dv\,du$$

$$= \frac{1}{3} \times 3 \times \frac{1}{2} \cdot 2^2 \times 1 + \frac{1}{6}3^2 \times \frac{1}{2}2^2 \times \log 2$$

$$= 2 + 3\log 2 \;\#$$